Dynamics of rotating fluids

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Studies of geostrophic turbulence, chaos and other non-linear phenomena in rotating fluids: the role of combined laboratory and numerical experiments

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Geostrophic turbulence occurs in many natural systems, including the Earth's atmosphere and oceans. So the non-linear dynamical processes involved are not only of interest to fluid dynamicists, but their study also bears directly on the interpretation of a wide range of natural phenomena and on technical problems encountered in practical meteorology and climatology, where a satisfactory theoretical basis for predictability is still lacking. In terms of resources, by far the greatest effort expended on geostrophic turbulence is made by meteorologists throughout the world, who carry out extensive programmes of observations of the atmosphere and related simulations, using some of the most powerful computers available for scientific research. Oceanographers are now joining in as they develop ambitious observational and computer-simulation programmes.

A complementary approach to the study of geostrophic turbulence and related non-linear flow phenomena in rotating fluids underlies the papers by Dr P.L. Read and Dr A.A. White, which the Editor has asked me to introduce. More modest than field studies in its demands on resources, the approach has proved extremely successful over a number of years. The work involved comprises systematic studies of the hydrodynamics of rapidly rotating fluids over a wide range of conditions, including laboratory systems and planetary atmospheres (using observations obtained by others from space missions). The use of laboratory systems is unusual in the investigation of large-scale natural systems, for in general it is impossible to achieve dynamical similarity, but in the case of geostrophic turbulence and other less chaotic but still essentially non-linear phenomena in rapidly rotating fluids, the key parameters governing the behaviour of large-scale systems *are* attainable in the laboratory. This discovery was amongst the main findings of laboratory experiments carried out by the writer and others well over thirty years ago, when transitive and intransitive flows, vacillation, multiple equilibria, etc. were first produced and studied. Through the work of leading theoreticians, these laboratory results have influenced studies of large-scale atmospheric motions, and they were seminal in certain developments in the basic mathematical theory of non-linear systems.

To paraphrase remarks of Lorenz (1967)*, by indicating the flow patterns that can occur and the conditions favourable to each, the initial experiments made possible the separation of essential from minor and irrelevant considerations in the theory of the global atmospheric circulation. They show, for instance, that while condensation of water vapour may play an essential role in the tropics, it appears to be no more than a modifying influence in temperate latitudes, because hydrodynamical phenomena found in the atmosphere, such as cyclones, jet streams and fronts, also occur in the laboratory apparatus where there is no analogue of the condensation process. Similar remarks apply to topographic features, which were intentionally omitted in the (initial) experiments. The so-called 'beta-effect' — the tendency for the relative vorticity to decrease in northward flow and increase in southward flow because of the variation with latitude of the Coriolis parameter — now appears to play a lesser role than had once been assumed. Certainly a numerical weather forecast would fail if the beta-effect were disregarded, but the beta-effect does not seem to be required for the production of typical atmospheric systems. The experiments have emphasized the necessity for truly quantitative considerations of planetary atmospheres. These considerations must, at the very least, be sufficient to place the Earth's atmosphere in one of the free non-axisymmetric regimes of thermal convection discovered in the laboratory work.

When the early experiments were carried out, available laboratory techniques were comparatively rudimentary and the role of computers very limited, owing to their ability to deal with nothing more complicated than the simplest flows encountered in the laboratory. Numerical modelling of these laboratory flows has made great strides in recent years, and the most powerful computers can now cope with many of the flow phenomena we are now able to produce and study in some detail in the laboratory, using the most advanced measuring systems we have been able to develop. The findings of these combined laboratory and numerical studies and associated analytical work continue to influence leading theoretical meteorologists and oceanographers, and they are now being applied systematically in diagnostic studies of the atmosphere and the testing of numerical models. So it is entirely appropriate that research of this kind should be carried out in meteorological institutions and its findings reported and discussed in journals such as the *Meteorological Magazine.*

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*Lorenz, E.N. *The nature and theory of the general circulation of the atmosphere*. Geneva, WMO No.218. TP.115, 1967.

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The dynamics of rotating fluids: the 'philosophy' of laboratory experiments and studies of the atmospheric general circulation

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Summary

It is instructive to consider the underlying basis for regarding certain laboratory experiments as contributing insight and information relevant to dynamical meteorology and oceanography. An intimately related question is 'what is meant by the term "model" in meteorology and oceanography, and what purpose does it serve?' The motivation and 'philosophy' behind such experiments are discussed by elucidating the general role of laboratory experiments and other models in fluid mechanics and in atmospheric dynamics. The thermally driven rotating annulus is examined as a particular example of relevance to general circulation studies in meteorology, and compared and contrasted with other experimental systems in fluid mechanics.

1. 'Models' in basic and applied science

In engineering and applied sciences, the term 'model' is commonly used to represent a device which imitates the behaviour of a physical system as closely as possible, but on a different (usually smaller) scale. The aim of such a model is normally to evaluate the behaviour of the physical system under practical conditions, for reasons connected with the exploitation of that system for economic, social, military or other purposes. Numerical weather prediction (NWP) models, for example, clearly fall into this category. By their very nature, such models are extremely complicated entities. Like the atmosphere itself, therefore, it is generally impossible to comprehend fully the complex interactions of physical processes and scales of motion that result in their synoptic behaviour. The success of these models can only really be judged by the accuracy of their predictions as directly verified against subsequent atmospheric observations. Climate models, on the other hand, are often comparable in complexity to those used for NWP, yet are frequently used in attempts to answer questions of economic, social or military importance for which little or no atmospheric data may be available to verify their conclusions (e.g. the CO_2 problem, the 'nuclear winter' debate, etc.).

In constructing such models and interpreting their results, it is necessary to make use of a different class of model — the 'conceptual' or 'theoretical' model — which may represent only a tiny subset of the processes active in the much larger, applications-oriented model, but whose behaviour may be completely understood (both qualitatively and quantitatively) from first principles. To arrive at such a complete level of understanding, however, it is usually necessary to make such models extremely simple in construction and highly idealized. An important prototype of such a model in fluid mechanics is that of dimensional or 'scale' analysis (a hybrid form of the technique was called 'inspectional analysis' by Birkhoff 1960), in which an entire problem is reduced to a determination of the essential balance of forces, and the consequent dependence of one or more observable (dimensionless) parameters on others in the form of power-law exponents. Following a systematic scale analysis, it is often possible to arrive at a scheme of approximations to the full mathematical description of a problem (e.g. the Navier–Stokes equations) which may then permit analytical solutions to be obtained. The quasi-geostrophic approximation is another important prototype (for example see Gill 1982) which enables a number of essential dynamical processes (e.g. barotropic and baroclinic instabilities, Rossby waves, 'free modes', etc.) to be studied in simplified (but nonetheless representative) forms.

For the basic researcher, such models are an essential device to aid and advance understanding. The latter is achievable because simple models enable theories and hypotheses to be formulated in a way

which may be tested (i.e. falsified, in the best traditions of the scientific method) against observations (e.g. of the atmosphere of the Earth and of other planets) and/or experiments. The ultimate aim of such studies in atmospheric science are an overall framework which sets in perspective all planetary atmospheres, of which the Earth's is but one example — see Fig. 1 (also see Lorenz 1967, Hide 1969, 1977, Hoskins 1983). Indeed this role is explicitly recognized in the World Climate Programme for example (see Lorenz 1967, World Meteorological Organization 1975).

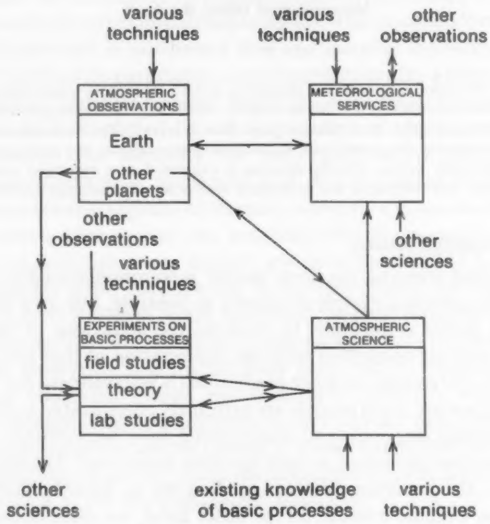

Figure 1. An attempt to illustrate the relationship between atmospheric science, applied meteorology and research on basic processes.

The role of laboratory fluid mechanics experiments in this scheme would seem to be as models firmly in the second category. Compared with the atmosphere, they are clearly much simpler in their geometry, boundary conditions and forcing processes (diabatic and mechanical). Their behaviour is therefore governed by a system of equations which can be stated exactly (i.e. no controversial parametrizations are necessary), although exact mathematical solutions may still be impossible to obtain. Unlike the atmosphere, however, it is possible to carry out controlled experiments in the laboratory to study dynamical processes in a real fluid without recourse to dubious approximations (necessary to analytical studies). For certain purposes, therefore, laboratory experiments can complement studies using complex numerical models, especially since (a) experiments have virtually infinite resolution compared with their numerical counterparts (subject only to the continuum hypothesis!) and (b) they are very cheap to run! In the context of simple analytical models of atmospheric processes, it is sometimes possible (though by no means automatically true) that a suitably designed laboratory experiment can be used to obtain a physical realization of that model in a real fluid, provided certain scaling assumptions (for 'dynamical similarity') can be satisfied (e.g. Birkhoff 1960). Such an experiment can be regarded as a 'test bed' for that model under highly controlled conditions.

In discussing the role of laboratory experiments, however, it is not entirely true to say that they have no direct role in the construction of more complex, applications-oriented models. Because the numerical

techniques used (finite-difference schemes etc.) in such models of the atmosphere can also be used to simulate flows in the laboratory under similar scaling assumptions, laboratory experiments can also be useful 'test beds' for directly verifying the accuracy of such techniques in a far more rigorous way than is possible using atmospheric data alone (see Hignett *et al.* 1985, White 1988).

2. General circulation studies and the rotating annulus

If the central problem concerning the global circulation of the Earth's atmosphere is that of 'predicting from the laws of classical physics that the atmosphere is necessarily organized as it is', then any approach towards obtaining such a prediction should include a minimal number of essential physical ingredients. At its most basic level, the general circulation is but one example of thermal convection due to impressed differential heating in the horizontal in a fluid of low viscosity and thermal conductivity. Laboratory experiments investigating such a problem should therefore include at least these factors, and be capable of satisfying scaling requirements for dynamical similarity to the relevant scales of motion in the atmosphere. Such experimental systems may then be regarded as representing the general circulation in the absence of various complexities associated, for example, with radiative transfer, atmospheric chemistry, boundary-layer turbulence, planetary curvature, topography, etc. (although some of the latter, such as planetary vorticity gradients and topography, can be included in a systematic way if required).

Experiments of this type are by no means a recent phenomenon, with examples published as long ago as the 19th century (e.g. Vettin 1884, Exner 1923, and see Fultz 1951 for a review of this early work). The modern development of experiments on the general topic of rotating fluids was begun by Taylor (1923), who also contributed greatly to the theoretical development of the subject. It was not until the late 1940s, however, that Fultz began a systematic series of experiments at the University of Chicago on rotating fluids subject to horizontal differential heating in an open cylinder (hence resulting in the obsolete term 'dishpan experiment'), and set the subject onto a firm footing (see Fultz *et al.* 1959). Independently and around the same time, Hide (1958) began his first series of experiments at the University of Cambridge on flows in a differentially heated rotating annulus, initially in the context of fluid motions in the Earth's liquid core. It is the latter system which is now considered in detail.

3. Flow regimes and transitions in the rotating annulus

The typical construction of the annulus is illustrated schematically in Fig. 2, and consists of a working fluid (usually a viscous liquid, such as water or silicone oil) contained in the annular gap between two coaxial, circular, thermally conducting cylinders, which can be rotated about their common (vertical) axis. The cylindrical sidewalls are maintained at constant but different temperatures (though see Read 1988) with a (usually horizontal) thermally insulating lower boundary and an upper boundary which is also thermally insulating and either rigid or free (i.e. without a lid).

Although a number of variations in these boundary conditions have been investigated experimentally, all such experiments are found to exhibit the same three main flow regimes, for example, as the rotation rate Ω varies. These consist of axisymmetric flow (in some respects analogous to Hadley flow in the Earth's tropics, and frequently referred to as the 'upper-symmetric regime'; see below) at very low Ω, regular waves at moderate Ω, and highly irregular, aperiodic flow at the highest values of Ω attainable (see Fig. 3). In addition, axisymmetric flows occur at all values of Ω at a sufficiently low temperature difference ΔT (a diffusively dominated regime termed 'lower symmetric' to distinguish it from the physically distinct 'upper-symmetric regime' mentioned above). The location of

these regimes are usually plotted on a 'regime diagram' with respect to the two most significant dimensionless parameters,

— a stability parameter or 'thermal Rossby number' $\Theta \equiv g\alpha\Delta Td/[\Omega(b-a)]^2$ providing a measure of the effect of buoyancy forces relative to Coriolis accelerations

— a Taylor number $\tau_a \equiv \Omega^2(b-a)^5/[v^2 d]$ measuring the effect of viscosity relative to Coriolis accelerations

where g is the acceleration due to gravity, α the thermal expansion coefficient of the fluid, v the kinematic viscosity and a, b and d are the dimensions indicated in Fig. 2. Fig. 4 shows a typical regime diagram with the locations of the regimes illustrated in Fig. 3 indicated.

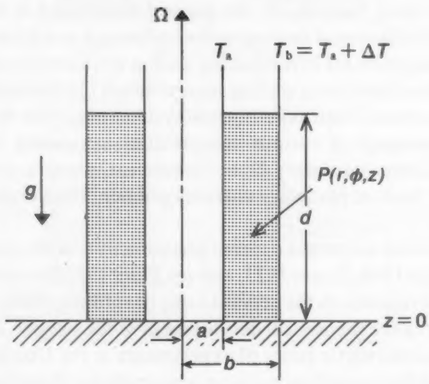

Figure 2. Schematic diagram of a rotating fluid annulus subject to a horizontal temperature gradient. P is a general point with polar coordinates (r,ϕ,z) in a frame of reference rotating with the apparatus, Ω is the angular velocity of basic rotation and g is acceleration due to gravity; region occupied by fluid is $a \leqq r \leqq b$, $0 \leqq z \leqq d$; T_a and T_b denote the respective temperatures of the cylindrical boundaries $r = a$ and $r = b$.

The regular waves may be either steady (apart from a slow drift) or 'vacillating' (i.e. with a periodic or nearly periodic time dependence). 'Amplitude vacillation' occurs in association with transitions towards a lower wave number (obtained by reducing Ω and/or increasing ΔT), and is characterized by periodic modulation of the wave amplitude and phase speed. 'Structural vacillation' (also known as 'shape' or 'tilted-trough vacillation') occurs as the irregular flow transition is approached, and is characterized by a nearly periodic tilting of the wave axis. This becomes more pronounced as Ω is increased, until the regular flow pattern breaks down into fully irregular flow. Another important property characteristic of the regular flow regime is that of intransitivity (i.e. multiple equilibrium states), in which two or more alternative flows with differing azimuthal wave number m can occur for a given set of parameters. The state obtained depends upon the initial conditions. In addition, transitions between different states in the regular regime, achieved by slowly changing the external parameters, often exhibit hysteresis, in that the location of a transition in parameter space depends upon the direction from which that transition is approached (e.g. $m = 3 \rightarrow 4$ does not occur at the same point as $m = 4 \rightarrow 3$). The latter properties are intimately connected with non-linear effects in the flow (for example see Pippard 1985) arising from the advection of heat and momentum in the fluid.

From a consideration of the conditions under which waves occur in the annulus (especially the location in parameter space of the 'upper-symmetric' transition, see Fig. 4) and a comparison with the

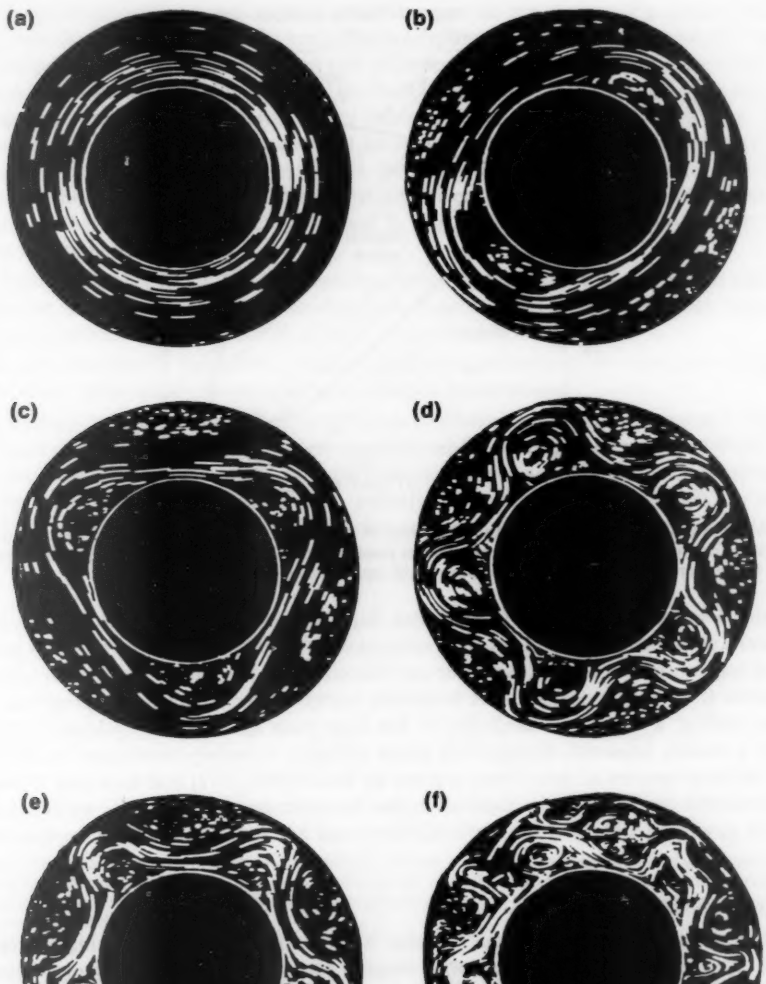

Figure 3. Streak photographs illustrating the dependence of flow type on rotation rate. Photographs were obtained in an annulus of $(b-a)=4.64$ cm, $d=13.5$ cm, $\Delta T=9$ K, in a working fluid of water/glycerol solution with density $\rho=1.037$ g cm^{-3}, and show the flow 0.5 cm below the free upper surface. Values of Ω and Θ are: (a) 0.41 rad s^{-1} and 7.3; (b) 1.07 rad^{-1} and 1.07; (c) 1.21 rad s^{-1} and 0.84; (d) 3.22 rad s^{-1} and 0.118; (e) 3.91 rad s^{-1} and 0.080; (f) 6.4 rad s^{-1} and 0.030.

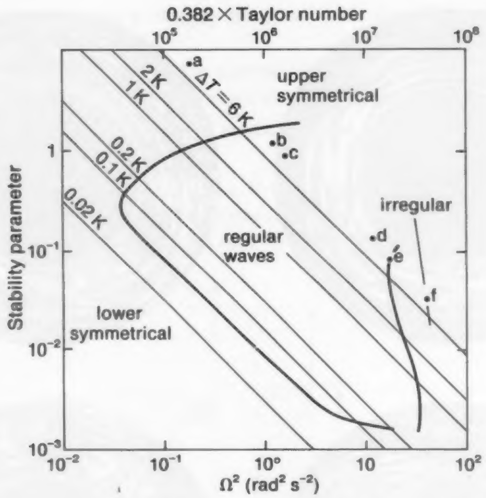

Figure 4. Typical regime diagram illustrating the dependence of the mode of convection on the two principal dimensionless parameters required to specify the system, namely a stability parameter and Taylor number (see text). The locations of the flows illustrated in Fig. 3 are shown as points a to f relative to the (approximate) regime boundaries.

results of linear instability theory, it is concluded that the waves in the annulus are fully developed manifestations of baroclinic instability (often referred to as 'sloping convection' from the geometry of typical fluid trajectories, for example see Hide and Mason 1975). Since these flows occur in the interior of the annulus (i.e. outside ageostrophic boundary layers) under conditions appropriate to quasi-geostrophic scaling, a dynamical similarity to the large-scale mid-latitude cyclones in the Earth's atmosphere is readily apparent, though with rather different boundary conditions. A more detailed discussion of the properties of these flows is given by Hide (1969, 1977) and Hide and Mason (1975). Associated with this conclusion is the implication that the waves develop in order to assist in the transfer of heat both upwards (enhancing the static stability) and horizontally down the impressed thermal gradient.

4. Other experimental systems

The regime structure for the thermal annulus is remarkable in exhibiting highly regular and predictable non-axisymmetric flows over a wide range of parameters. If such a regime structure were to apply generally to any fluid system, it could have important implications for theories of atmospheric predictability. Evidence for regular flow regimes in systems more closely akin to planetary atmospheres is currently sparse (for example see James and Gray 1983), although the atmospheres of Mars and Jupiter display some intriguing examples of highly persistent and regular features (for example see Leovy 1979). It is of interest, therefore, to compare the regime structure of the thermally-driven annulus with that of other fluid systems in the laboratory which investigate quite different dynamical processes.

4.1 *The two-layer annulus or open cylinder*

Another system which exhibits baroclinic instability in a different form to that of the thermal annulus is found in the rotating, two-layer experiment (for example see Hart 1979). Two immiscible liquids of

differing density are placed in an open circular cylinder or coaxial annulus (see Fig. 5(a)) which can, like the thermal annulus, be rotated about its vertical axis of symmetry. Motions are driven by rotating the rigid upper boundary of the fluid at a different rate to the rest of the apparatus, imparting a vertical shear which causes an axisymmetric deformation of the fluid interface (thereby storing potential energy in a way analogous to the sloping isotherms in the thermal annulus). Baroclinic instability occurs via non-axisymmetric deformations of that interface, thereby transferring angular momentum (rather than heat). A significant advantage of this system in the study of sloping convection is that it is more amenable to mathematical analysis than the thermally driven system (for example see Pedlosky 1979).

The regime diagram is schematically shown in Fig. 5(b), plotted in terms of the following dimensionless parameters:

- a Rossby number $Ro \equiv \Delta\Omega/2\Omega$ (usually plotted as the inverse of Ro) which measures the relative importance of inertial accelerations due to the imposed shear with respect to the Coriolis accelerations associated with the basic rotation
- a Froude number $Fr \equiv \rho_0(2\Omega b)^2/(g\Delta\rho d)$ (roughly equivalent to $1/\Theta$ — see section 3) measuring the effect of buoyancy forces relative to Coriolis accelerations

where Ω is the basic rotation, $\Delta\Omega$ the difference in rotation between the main apparatus and the lid and $\Delta\rho_0$ is the mean density. At low Fr and/or very small Ro, all flows are axisymmetric, while at $Fr>5$ with moderate Ro, waves are found to occur. As in the thermally driven annulus, these waves are regular and steady at moderate values of Fr, and undergo periodic modulations at larger values. At the highest values of Fr attainable the flow becomes irregular and aperiodic, much as observed in the thermal annulus at low values of Θ.

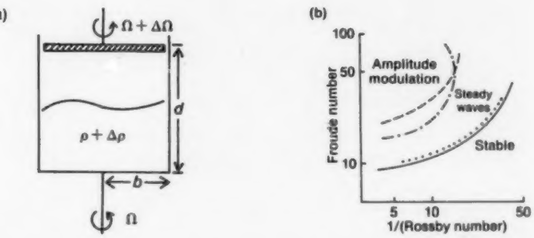

Figure 5. (a) Schematic diagram of the two-layer baroclinic system (shown for an open cylinder). (b) Regime diagram for co-rotating two-layer flow. Continuous line shows the observed neutral curve, dotted line shows a theoretically predicted neutral curve, dash-dotted line shows the transition to amplitude vacillation, while the dashed line shows a prediction from a spectral numerical model (see Hart 1979).

4.2 *Rayleigh–Bénard convection*

The properties of thermal convection without rotation in the presence of an unstable thermal gradient in the vertical have been studied for many years (for example see Swinney and Gollub 1985 for a review). The regime structure is found to depend significantly upon the aspect ratio D/L (where D and L are vertical and horizontal length scales respectively). Fig. 6 shows a typical regime diagram for low aspect-ratio systems (from Krishnamurti 1973), which depends mainly upon the following:

- a Rayleigh number $Ra \equiv g\alpha\Delta TD^3/kv$ measuring the importance of buoyancy forces with respect to diffusion associated with viscosity and thermal conduction
- a Prandtl number $Pr \equiv v/k$ which measures the relative importance of viscous to conductive diffusion

where k is the thermal diffusivity and v the kinematic viscosity. No convection occurs at all when $Ra<Ra_c$, where Ra_c is a critical or transitional value of Ra, typically ~ 1500. When $Ra>Ra_c$, convection begins in the form of steady, two-dimensional rolls; the precise form and orientation of these depend upon the lateral boundaries. As Ra continues to increase, the rolls give way first (at high Pr) to steady, three-dimensional cells, then to time-dependent flows, which can be regular and periodic (especially at low Pr), before finally becoming irregular and turbulent. For large aspect-ratio systems, there is little evidence for regular behaviour at the onset of convection, with the flow rapidly becoming irregular and turbulent (with the development of plumes etc.).

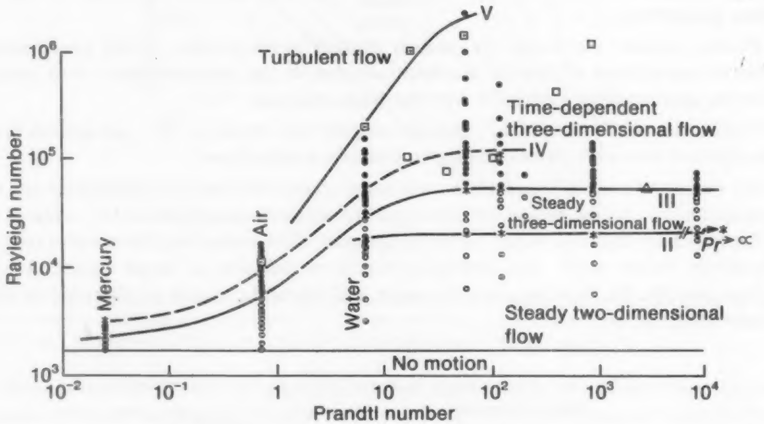

Figure 6. Regime diagram for low aspect-ratio Rayleigh–Bénard convection as a function of Rayleigh and Prandtl numbers (from Krishnamurti 1973).

4.3 *Taylor–Couette flow*

The instability of a homogeneous fluid subject to mechanical shear at its boundaries is another classical problem in fluid mechanics, and was extensively studied (both theoretically and experimentally) by Taylor (1923). To study the flow in the laboratory, the fluid is usually contained in an annulus of narrow gap width $(b-a)$ and large depth, in which the inner and outer cylindrical sidewalls may be rotated independently (at Ω_a and Ω_b — see Fig. 7(a)). The flow obtained depends principally upon Reynolds or Taylor numbers measuring the relative importance of viscous and Coriolis forces near each side boundary, and defined by Ω_a, Ω_b, a and b (i.e. $Re_a \equiv \Omega_a a^2/v$ and $Re_b \equiv \Omega_b b^2/v$) and the main aspect ratios of the apparatus. Because the experimental arrangement is simple and easy to control (and has applications connected with the lubrication of bearings), it has received wide attention, especially in recent years in connection with the rich behaviour observed in its transitions to turbulent flow (for example see Swinney and Gollub 1985 for a review).

At low values of Re_a (keeping Re_b constant and positive) the flow is uniform throughout the apparatus, in response to the imposed shear. Above a critical value of Re_a (related to the Rayleigh criterion for inviscid centrifugal instability, e.g. Greenspan 1968), the flow develops a series of steady axisymmetric rolls which are periodic along the rotation axis (Taylor vortices). As Re_a is increased further, the rolls develop waves in the azimuthal direction ('wavy vortex flow'), and the flow becomes doubly-periodic (see Fig. 7(b) — adapted from Andereck *et al.* 1986). At much higher values of Re_a,

further instabilities occur until fully turbulent flow is obtained, although the detailed sequence of events is highly complicated (the sequences are discussed in some detail by Andereck *et al.* 1986). Intransitivity in the number of Taylor vortices obtained at a given set of parameters in the regular flow regime is often exhibited much more strongly in the Taylor–Couette system than in the thermal annulus, in the sense that many more distinct states may be obtained for a given set of parameters.

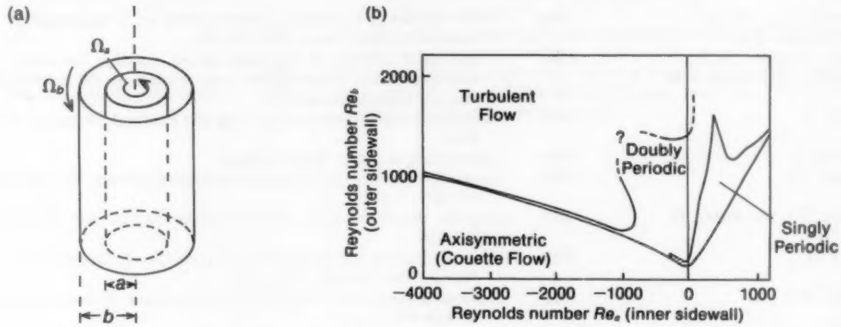

Figure 7. (a) Schematic diagram of the Taylor–Couette system, and (b) regime diagram as a function of Re_a and Re_b near the onset of Taylor vortices (adapted from Andereck *et al.* 1986).

5. 'Universal' behaviour and dynamical systems

The above examples serve to demonstrate (and the list is by no means exhaustive) that the presence of steady symmetric, regular periodic, and irregular flow regimes are the norm rather than the exception in fluid mechanics (at least for systems characterized by a degree of spatial symmetry in their boundary conditions). Some justification for this conclusion has recently emerged from studies of the general theory of non-linear dynamical systems, of which fluid flows may be but a single example (for example see Cvitanovic 1984).

A dynamical system may be loosely defined as one whose state is fully determined by its position in a suitably-defined phase-space, and whose evolution in time (i.e. its first time derivative) depends solely upon its current position in phase-space (i.e. not on external random forcing etc.). Numerous other examples of dynamical systems are found outside hydrodynamics, including non-linear optics, electronics, engineering structures, chemical reactions, and even certain processes in living organisms (for example see Cvitanovic 1984, Pippard 1985 for reviews). Of particular interest has been the identification of a number of kinds of characteristic sequence by which systems may undergo transitions ('bifurcations') from steady to irregular behaviour via regular, periodic states ('routes to turbulence', for example see Eckmann 1981), and in which irregular behaviour may initially involve only a relatively small number of the available degrees of freedom in a way first identified by Lorenz (1963) — 'deterministic chaos'. The latter is in complete contrast to earlier classical theories of the transition to turbulence (Landau and Lifshitz 1959), in which turbulence and irregular flow were associated with the excitation of a very large number of degrees of freedom.

The possible applications of these theoretical developments to real systems continues to be an active area of research activity. Much theoretical and experimental effort is currently being devoted

(a) to attempts to determine the range of possible 'routes to turbulence' and their 'universal' properties (where the term 'universal' is used in a similar sense to that used in the study of phase transitions and critical phenomena (for example see Wilson 1979), and

(b) to find the optimum ways of characterizing regular and chaotic (aperiodic) behaviour in experimental (for example see Eckmann 1981, Procaccia 1985, Eckmann and Ruelle 1985, Mayer-Kress 1986, Bell *et al.* 1986) and even certain natural systems (including climate, for example see Nicolis and Nicolis 1984, Fraedrich 1986, Grassberger 1986).

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The dynamics of rotating fluids: the internally heated annulus

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Summary

Direct internal heating of a rotating fluid annulus in the laboratory, while cooling at both sidewalls, leads to profound changes in flow structure associated with the resulting non-monotonic horizontal temperature gradient. The basic flow regimes and heat transfer properties of such flows, however, are largely unchanged from the more conventional annulus experiments which impose differential heating and cooling only at the sidewalls. Regular waves occur in the internally heated system, but take the form of compact, nearly isolated closed eddies, embedded in a mean zonal flow with strong lateral shear. The properties of the flows in the regular wave regime of the internally heated annulus are compared with those of large-scale eddies in the atmospheres of Jupiter and Saturn, and a possible analogy between the two systems is explored. The implications of such an analogy are discussed with particular reference to the role of the large-scale eddies in the atmospheres of the major planets, and theories of atmospheric predictability.

1. Introduction

In carrying out experiments in the laboratory on thermal convection in a rotating, baroclinic fluid, it is of great importance to establish which aspects of the flows observed are generally applicable to any fluid system, and which may be more specific to the particular system under study (e.g. dependent on particular means of forcing or boundary conditions). One way in which this may be investigated is by incorporating variations, e.g. into the construction of the experiment, the fluid properties and especially the boundary conditions. Here we consider some effects of variations in the thermal boundary conditions and the overall distribution of heating and cooling in the annulus. Thus, for example, by using direct internal heating of the working fluid, it is possible to investigate the properties of baroclinic waves in a zonal flow for which the horizontal thermal gradient is non-monotonic, and therefore much removed from the kind of flow considered in 'classical' theoretical analyses. A further motivation for

studying such flows has arisen recently from observations of the atmospheres of other planets, whose composition, scale and means of thermal and mechanical forcing (and their spatial distributions) may be quite different from those of the Earth. The atmospheres of Jupiter and Saturn offer some particularly intriguing phenomena which could be manifestations of sloping convection under conditions more closely similar to those which can be obtained in the internally heated annulus (for example see Hide 1981, Read and Hide 1983, 1984, Read 1986a). This potential application of the annulus experiments will be considered in sections 3 and 4.

2. Baroclinic waves with internal heating

The laboratory system considered is an annulus of conventional design, but in which heating may be applied internally via ohmic dissipation of an alternating electric current passed through the working fluid (which is a weak electrolyte) between the (electrically-conducting) sidewalls. Either or both sidewalls can be cooled in the usual way, allowing a wide variation in the effective thermal boundary conditions and distribution of heat sources and sinks (for more details see Hide and Mason 1970, 1975, Read 1986b).

The various regimes of flow obtained in such a system are found to be very similar to those in the conventional (wall-heated) annulus, with transitions between them occurring under comparable experimental conditions (again measured by a stability parameter Θ and Taylor number τ_a (see Read 1988) almost regardless of the distribution of cooling at the boundaries (for example see Hide and Mason 1970). Thus, axisymmetric flows occur at the lowest rotation rates Ω, regular waves at intermediate Ω and irregular waves at the highest values of Ω. The most significant difference between the behaviour of internally heated flows to those in the conventional annulus appears to be in the form of periodic 'vacillations' — recent work suggests that 'amplitude vacillation' rarely occurs in the internally heated annulus, while 'wave-number vacillation' is the preferred form of structural vacillation. A selection of regular and irregular wave flows for cases in which both sidewalls are at the same temperature ($\Delta T = 0$) are shown in Fig. 1.

The form of the horizontal flow pattern varies considerably in the axisymmetric and regular wave regimes, depending upon the thermal boundary conditions at the sidewalls (see Fig. 2). Internal heating forces upward-motion throughout the interior, which must match onto the horizontal boundary layers via the Ekman suction condition. Thus, the relative vorticity of the axisymmetric or mean zonal flow is anticyclonic at upper levels and cyclonic at lower levels (a more detailed analysis is given by Quon 1977, Read 1986b). In the presence of non-axisymmetric waves, this results in trains of eddies which are predominantly anticyclonic at upper levels, but with an associated meandering jet stream whose location and strength depends upon the thermal boundary conditions at the sidewalls, and hence upon the net inward or outward radial flux of heat. This may be approximately related to the mean zonal flow near the side boundaries (where the eddies are weak) by

$$H(r, t) = -(v/\Omega)^{\frac{1}{2}} [\bar{T}(r, z=d, t) - \bar{T}(r, z=0, t)]\, \bar{u}(r, z=d, t) \quad \dots \dots \dots \quad (1)$$

(see Hide and Mason 1970, 1975, Hide 1981), where H is the total heat flux through a cylindrical surface of height d and radius r, $\bar{T}(r, z, t)$ and $\bar{u}(r, z, t)$ are the mean zonal temperature and zonal velocity respectively, and v is the kinematic viscosity. Note that the terms multiplying $\bar{u}$ are negative definite, so that $\bar{u}$ may be seen to reflect directly the partition of heat flow between the inner and outer sidewalls —the result is summarized schematically in Fig. 3 for comparison with the examples in Fig. 2.

The most striking example occurs when $\Delta T = 0$, for which heat is removed at equal rates at both sidewalls. The axisymmetric flow then consists of a temperature maximum at mid-radius on horizontal

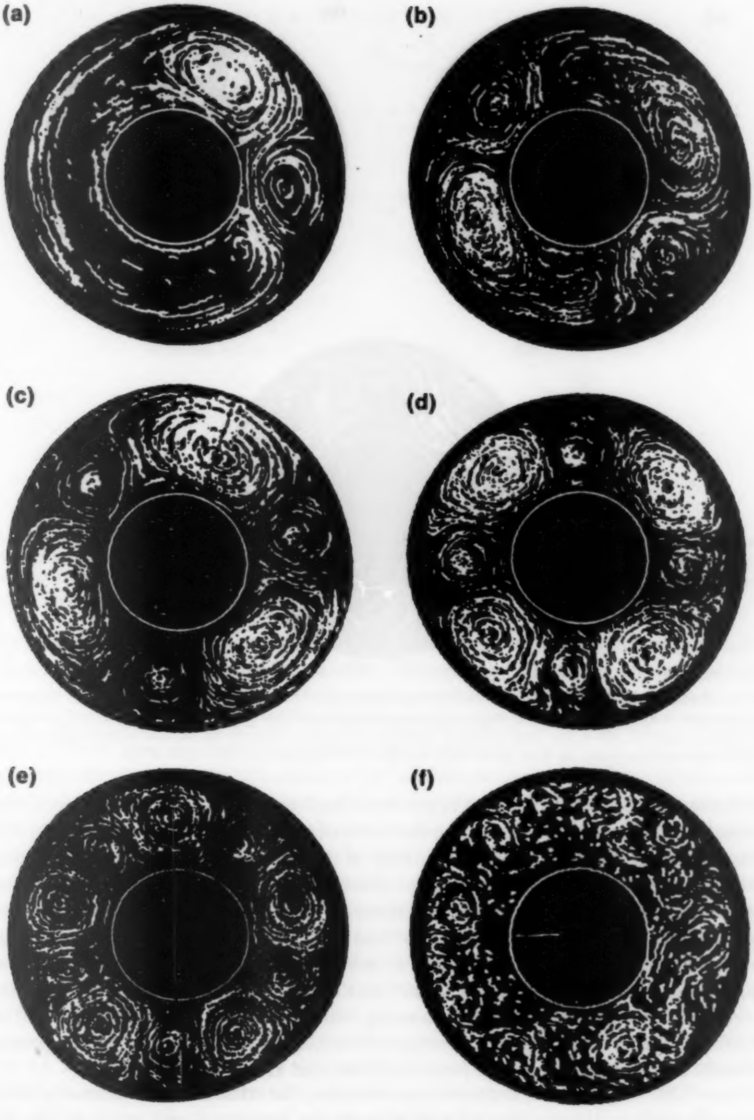

Figure 1. Streak photographs illustrating the dependence of flow type in an internally heated annulus on rotation rate. Photographs were obtained in an annulus of (radius b − radius a) 6.02 cm, cylinder height 16.1 cm, power input 100 W (except for (a) and (b) which were at 750 W and 400 W respectively), with a solution of water/glycerol as working fluid with density $\rho = 1.043\ \mathrm{g\,cm^{-3}}$, and show the flow 4.5 cm below the (rigid, non-slip) upper surface. Values of stability parameter Θ are: (a) 2.92, (b) 2.84, (c) 2.16, (d) 0.72, (e) 0.48 and (f) 0.09.

Figure 2. Streak photographs illustrating the dependence of upper-level flow pattern on the form of the impressed horizontal temperature gradient in a rotating annulus subject to internal and sidewall heating. Flows were obtained in the same apparatus as Fig. 1 at a power input of 100 W but with different values of ΔT (hence varying the partitioning of heat extraction between inner and outer sidewalls) namely (a) $T_b - T_a = -3$ K, (b) $T_b - T_a = 0$ K and (c) $T_b - T_a = +3$ K.

surfaces, with strong anticyclonic shear at upper levels between two opposing jet streams (and cyclonic shear at lower levels). Baroclinic waves take the form of trains of compact, apparently isolated oval eddies, all circulating in the same sense as the shear of the mean zonal flow (i.e. anticyclonic at upper levels). Little motion appears outside the eddies themselves, apart from a very weak meandering jet stream. The isolated appearance of the eddies is most obvious for the lowest wave numbers (m) which occur at high values of Taylor number and Θ. Fig. 1(a) shows the upper-level flow for a typical $m = 1$, in which the eddy is seen to be concentrated into a narrow range in azimuth. Although the eddy has a 'solitary' appearance, its structure is not consistent with conventional 'soliton' or 'modon' solutions, but rather has the form of a wave packet encompassing little more than a single wavelength in azimuth. Non-linear effects are therefore of great importance in setting up the flow, strongly steepening the basic wave in azimuth. Further details may be found in Read and Hide (1984).

Like the baroclinic waves in the conventional annulus, the eddies in the internally heated system contribute significantly to the transfer of heat in both the horizontal and vertical. Fig. 4 shows the variation of Nusselt number (a dimensionless measure of total heat transfer, normalized by what would occur via molecular conduction only; see Hide and Mason 1975) with rotation rate for the internally heated system, as measured in the laboratory (note that Nusselt number for an internally heated system is defined slightly differently than for a boundary-heated flow — see Hide and Mason 1970, Read

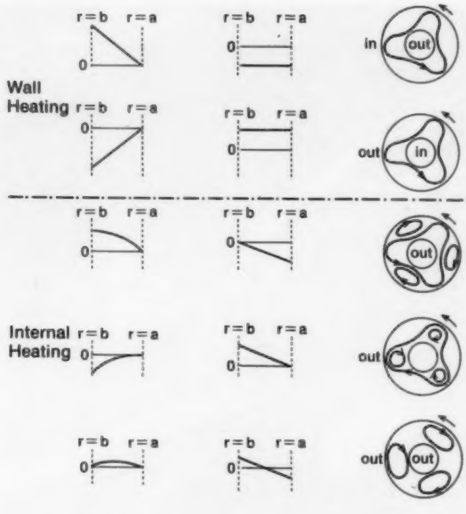

Figure 3. Schematic illustrations of the variations of the impressed temperature gradient with radius r (left column); $d\bar{T}/dr$, proportional to minus the top-surface zonal velocity (see equation (1)) (centre column); and the main characteristics of the top-surface flow pattern in the steady-wave regime based on the integral constraints expressed in equation (1) (right column) — see text and Hide and Mason (1970, 1975).

1986b). The Nusselt number which would occur if the flow were to remain axisymmetric is also shown (derived from an axisymmetric numerical model). The presence of regular eddies maintains the heat transfer close to the level in the absence of rotation until the flow becomes irregular.

Recent (as yet unpublished) studies in the Geophysical Fluid Dynamics Laboratory of the Meteorological Office have incorporated a 'planetary vorticity gradient' (similar to a β-effect) by using sloping endwalls, thereby allowing the depth of the annulus to vary with radius. Similar methods were investigated in the conventional annulus by Mason (1975). The effect of sloping boundaries on the internally heated flows is found to be very similar to that on other annulus flows, in introducing wave dispersion in a way qualitatively similar to that of Rossby waves — eddies drift with respect to the mean zonal flow at a rate inversely proportional to their wave number and proportional to the effective β ($\propto \Omega$). At higher rotation rates, the radial scale of the eddies may be reduced, so that eddies no longer fill the annulus gap. This may ultimately result in two (or more?) independent, parallel trains of waves and eddies adjacent to each sidewall, associated with a series of parallel zonal jet streams.

3. Eddies on Jupiter and Saturn

Jupiter and Saturn lie well beyond the Earth's orbit at mean solar distances of 5.2 and 9.6 AU respectively (1 AU = mean Earth–Sun distance = 1.5×10^{11} m). They are giant planets with radii $\approx 7.14 \times 10^4$ km and 6.03×10^4 km respectively, but consist largely of hydrogen (90%) and helium (10%) with small rocky cores. Both planets rotate rapidly (sidereal periods of 9h 55m and 10h 39m respectively) and are shrouded in dense cloud decks consisting mainly of ammonia ice.

Both planets exhibit a variety of eddy-like features at the level of the upper cloud-decks, some of which appear to persist for very long periods (from months to many years). The best known examples are the Great Red Spot (GRS) on Jupiter — discovered in the 17th century — and the White Ovals (also

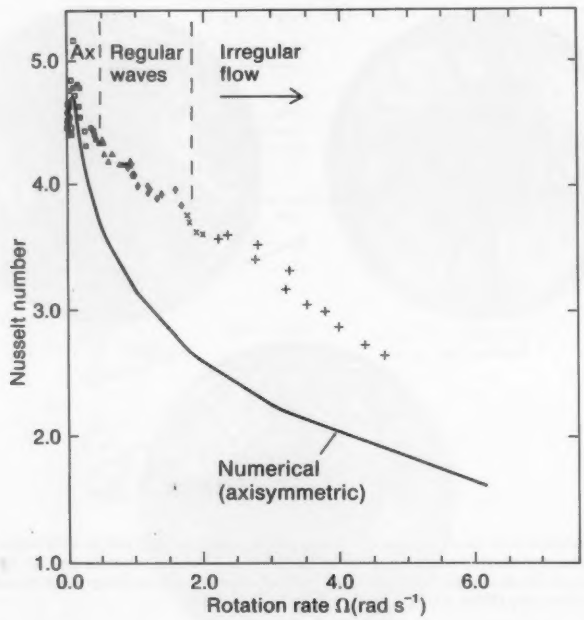

Figure 4. Variation of Nusselt number (a measure of the total heat transfer with respect to that due to conduction alone) for an internally heated annulus as a function of rotation rate Ω. Results shown are laboratory measurements by Read (1986b and unpublished) for various baroclinic wave numbers, compared with the axisymmetric heat transfer over the same range of Ω, obtained from an axisymmetric numerical model (Read 1986b). The results shown are (□) laboratory (axisymmetric), (△) laboratory (wave number $m=3$), (◇) laboratory ($m=4$), (X) laboratory ($m=4$ to 6, wave number vacillation), (+) laboratory (irregular flows) and (——) numerical (axisymmetric).

on Jupiter) — observed to form in 1939. Some examples are illustrated in Fig. 5. Most long-lived eddies on Jupiter and Saturn take the form of oval spots of various colours, the most common of which are found in regions of anticyclonic mean zonal shear between pairs of opposing mid-latitude jet streams, and are characterized by anticyclonic circulation at the upper cloud-levels. They include the GRS, the White Ovals and other smaller white spots at higher northern and southern latitudes on Jupiter, and some similar brown spots on Saturn. Cyclonic examples (occurring in regions of cyclonic mean zonal shear) include the brown 'barges' on Jupiter. An intriguing property of many of the smaller anticyclonic ovals on Jupiter is that they appear in almost regularly spaced trains in longitude, often interspersed with weaker cyclonic circulations.

Such features do not bear much resemblance to the more familiar baroclinic eddies in the Earth's atmosphere and in the conventional annulus experiments, and accordingly many different suggestions have been made to account for their nature and various properties (see Ingersoll 1981, Ingersoll *et al.* 1984, Read 1986a, Williams 1986 for reviews). The resemblance between these features and the baroclinic eddies obtained in the internally heated annulus, however, would suggest that sloping convection was a strong candidate to account for their nature and origin. Unfortunately, for various reasons the eddy-like features in all these different models are difficult to compare dynamically with the Jovian and Saturnian eddies:

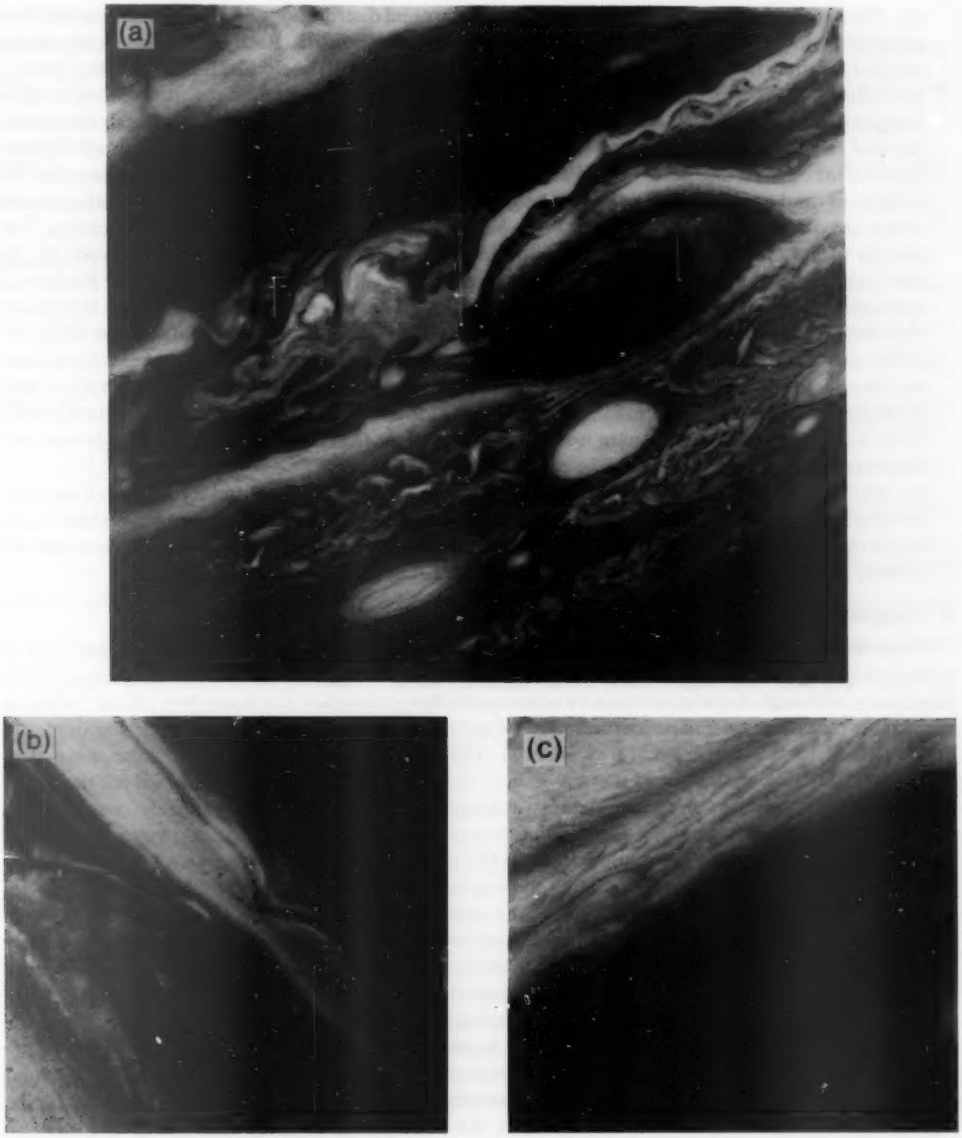

Figure 5. Some examples of long-lived oval eddies in the atmospheres of Jupiter and Saturn. (a) Voyager 2 image of Jupiter in the vicinity of the Great Red Spot (GRS), showing the GRS itself (centre right), a White Oval (just below the GRS) and other smaller features, (b) Brown Spot 1 at latitude 42.5° N on Saturn from Voyager 2, and (c) Voyager 1 image of a long-lived cyclonic 'barge'; at latitude 15° N on Jupiter.

(a) The appropriate boundary conditions and background distributions of wind and temperature are probably rather different for Jupiter and Saturn than for the Earth. Both major planets are almost certainly entirely fluid throughout, with no solid surface beneath the clouds. Both generate at least as much heat in their deep interiors as they receive from the Sun. This has the consequence that the deep interior is likely to be nearly isentropic and very weakly stratified, also causing large-scale horizontal thermal contrasts to be very small (see Flasar 1986 for a review). The observed thermal contrast between the equator and poles on Jupiter and Saturn at the cloud tops is very small. At least near the cloud tops (and probably some distance below — see Flasar 1986) the large-scale thermal contrasts which are observed are associated with the banded structure of the zonal winds and clouds. The pattern of zonal winds are consistent with an application of the thermal-wind equation to the observed thermal contrasts with latitude, which are actually non-monotonic (i.e. the horizontal thermal gradient reverses several times between the equator and poles — see the laboratory experiments described in section 2). More recent analyses of spacecraft infra-red data by Gierasch *et al.* (1986) confirm an association between the pattern of zonal winds and clouds and a thermally-driven meridional circulation with upwelling in regions of anticyclonic shear (again see the internally heated experiments mentioned in section 2). Many aspects of the deep structure of the flow and the effective lower boundary conditions remain uncertain, however, so that any hypothesis for the Jovian and Saturnian eddies must continue to be controversial.

(b) The available atmospheric data refer only to a thin layer around the cloud tops. It is not possible, therefore, to determine many useful dynamical diagnostics with which to test theories (indeed this is one of the major challenges posed by Jupiter and Saturn to dynamicists — to design an observational strategy which could yield conclusive results, e.g. see Allison and Stone 1983).

4. Long-lived eddies on Jupiter and Saturn as sloping convection?

Given that the relevant similarity parameters (e.g. Rossby, Richardson and Burger numbers) are of comparable magnitude for the long-lived Jovian and Saturnian eddies and the baroclinic eddies in the laboratory (although disputes continue concerning the Burger number, for example see Read 1986a), it is plausible that the atmospheric features may indeed represent a form of sloping convection similar to that obtained in the internally heated annulus. The detailed resemblance between the mean flow structures, eddy flow patterns and thermal structures (so far as they can be determined for the major planets) serve to support such an analogy, although these factors are not entirely conclusive in themselves. The relevance of quasi-geostrophic 'free modes' to the laboratory flows (for example Read 1985, White 1986) may also be of importance in this context, since similar theoretical solutions form significant components of many competing theories of the Jovian features (see Read 1986a). Further observations tailored towards answering the many questions which arise from this hypothesis are clearly required, almost certainly necessitating data from new spacecraft missions in due course (currently planned missions include 'Galileo' orbiter and probe to Jupiter and the Hubble Space Telescope — both seriously delayed by the recent Space Shuttle disaster!).

Nonetheless, the available observations do demonstrate that eddies occur in the atmosphere of the major planets which are characterized by a high degree of symmetry in the spatial organization of the flow, and long persistence times despite the presence of chaotic small-scale features in the background flow. Laboratory studies have indicated the importance of non-linear advection in helping to sustain the regular flows, which would seem to contradict the conclusions of some theories of atmospheric predictability concerning the role of advection in promoting the breakdown of an initial pattern of flow into irregular chaotic motion. Should it ultimately be demonstrated that the long-lived eddies on Jupiter and Saturn are manifestations of sloping convection, it would serve to confirm the latter process as an important paradigm for large-scale flows in planetary atmospheres.

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The dynamics of rotating fluids: numerical modelling of annulus flows

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Summary

Comparing numerically simulated annulus flows with good quality laboratory measurements is an invaluable way of testing the dynamical formulations of numerical models. Some qualitative and quantitative comparisons of this type are described, the model being a grid-point Navier–Stokes solver and the measurements being of temperature, horizontal velocity components and total heat flux. Good agreement is found in many instances. A case in which serious discrepancies occur is investigated in detail: it is found that the numerical model generates spurious eddy motion at intermediate resolution, but not at low or high resolution. Comparison of model results with published 'bench-mark' numerical solutions for flow in a square cavity reveals a high level of agreement, and suggests the feasibility of using annulus measurements to verify numerical models in engineering as well as meteorological contexts. Some closing comments are made on the extent to which the various flow phenomena of the annulus are accounted for by existing theoretical models.

1. Introduction

Experience suggests that many meteorologists consider laboratory work on rotating flows to be a rather academic activity — to be regarded, perhaps, with polite indifference. To such sceptics, numerical modelling of laboratory flows may seem doubly removed from everyday meteorological reality. Read (1988) has laid out the practical and scientific grounds for the laboratory work. Here the equally strong justification for the associated numerical simulations will be given, and a selection of results presented. Some theoretical aspects will also be briefly discussed.

2. Why model annulus flow?

There are two main justifications:

(a) Numerical-model verification — comparison of results with good laboratory measurements affords an invaluable test of numerical formulations.

(b) Scientific investigation — good numerical simulations provide much data whose analysis may greatly assist understanding of the real flows.

Of these, (a) is the more important in practical meteorological respects (at least in the short or medium term) and it is this which will be dealt with here. In the atmosphere the forcing processes are many and various. Momentum sources and sinks arise from various types of motion — such as small-scale turbulence, cumulus convection, cumulonimbus and some gravity waves — that are not explicitly resolved in large-scale numerical models. Diabatic heating and cooling occur as a result of sub-gridscale motions (see above), radiative effects and latent heat release/absorption in condensation/evaporation. All these processes are highly variable in space and time, and their representation or 'parametrization' in weather forecasting and climate models is a matter of considerable complexity and uncertainty.

On the other hand, in the rotating laboratory-annulus the only significant forcing processes are molecular viscosity and conductivity, which can be represented to high accuracy using established formulae; over wide ranges of conditions all flow scales can be resolved by tractable grids. Thus no 'parametrizations' are required in numerical simulations. Comparison with experimental measurements therefore enables the dynamical formulation of numerical models to be tested ('verified') to an extent which is virtually impossible through comparison with meteorological data because of the uncertainties concerning the atmospheric forcing processes.

3. Simulations of non-axisymmetric annulus flows with a 16-level model

In the familiar 'wall-heated' annulus system, concentric cylinders are maintained at different temperatures and the upper and lower bounding surfaces are horizontal and rigid (see Fig. 2 of Read 1988). A numerical model — the 'Met O 21 model' — of time dependent, non-axisymmetric flow in this system has been in use for several years within the Meteorological Office. It is a grid-point formulation based on the (non-hydrostatic) Navier–Stokes equations for incompressible baroclinic flow. Standard resolution is 16 (vertical) × 16 (radial) × 64 (zonal) points; the grid is stretched in the vertical/radial plane so that boundary layers are resolved adequately. Conservative finite difference schemes are used. Details are given by James *et al.* (1981) and Hignett *et al.* (1985). The model is similar to that described and operated by Williams (1969, 1971) and has been developed from programs originally constructed by Dr S.A. Piacsek and co-workers at the US Naval Research Laboratories, Washington DC. It is at least as tightly formulated dynamically as the typical primitive equation weather forecasting or global climate model and it runs on the Cyber 205 computer. Comparisons of the flow simulations with laboratory measurements may be made at a qualitative or a quantitative level.

3.1 *Qualitative comparisons*

The main flow phenomena of the wall-heated rotating annulus are axisymmetric flow, steady waves, intransitivity, wave-number transitions, hysteresis, amplitude vacillation, structural vacillation and irregular flow (see Read 1988). All of these have been produced by the Met O 21 model — although little attention has been paid as yet to structural vacillation and irregular flow. It should be noted also that the conditions (of rotation rate, temperature difference, etc.) under which a given flow phenomenon occurs in the simulations (at standard resolution) are in some cases noticeably different from those under which it occurs in the laboratory. However, the model is clearly capable of producing the phenomena that are observed, and this in itself is important information. Hignett *et al.* (1985) give a detailed survey. As examples of their results, Fig. 1 shows some upper-level pressure fields and a kinetic energy diagram from a simulated amplitude vacillation.

3.2 *Quantitative comparisons*

In the laboratory annulus, temperatures are measured at mid-radius and mid-depth using a ring of 32 thermocouple junctions. Total heat flux by the working fluid is calculated from the inflow and outflow temperatures of the water circulating within the inner (cool) wall. (Ideally, this measurement is made when the ring of thermocouple junctions is absent.) In a physically separate apparatus having the same dimensions, horizontal velocity components are obtained at five levels using a particle tracking technique.

Hignett *et al.* (1985) describe a quantitative comparison of a simulated steady wave flow with measurements made using these techniques. Some of their results are presented in Table I. The simulated main-wave phase speed is in very good agreement with the value calculated from the velocity measurements. Main-wave temperature amplitudes agree to better than 5% and maximum zonal mean flows differ by only about 10%. In fact, Hignett *et al.* found good agreement between simulated and measured flows as regards both the zonal mean and main-wave fields at each of the five levels of measurement. The comparison provides strong evidence that primitive equation numerical models can give good simulations of quasi-geostrophic wave motion (see Read 1988) in rotating, baroclinic fluids.

4. High-resolution simulations of axisymmetric annulus flows

An important aspect of any numerical simulation is the dependence of results on spatial resolution. As a step towards understanding the convergence (or otherwise!) of the annulus simulations as the mean

grid-length is reduced, two series of high-resolution axisymmetric flow integrations have recently been undertaken. (The storage requirements of axisymmetric flow simulation are much less than those of the non-axisymmetric case, and so fine resolutions can be readily investigated. The work has been done on the IBM 3081 computer.) The integrations offer encouraging evidence of the capacity of numerical models to give accurate results. Some of them, however, give insight into model pathology.

4.1 *Heat flux comparisons*

Special emphasis will be placed here on the total heat flux across the system in its steady state. This quantity may be accurately measured in the laboratory without the use of probes which might disturb

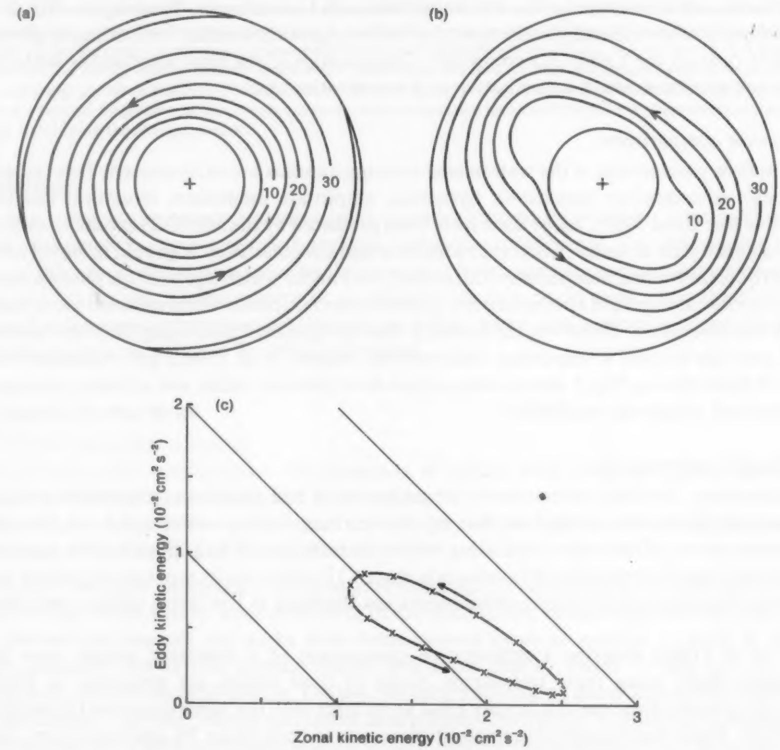

Figure 1. Numerical simulation of amplitude vacillation. (a) and (b) show pressure fields at an upper level in the flow at minimum and maximum wave amplitude respectively. Arrows indicate the direction of geostrophic flow. Quantity plotted is deviation of pressure/mean density from an arbitrary reference value; units are 10^{-1} cm^2 s^{-2}. The amplitude vacillation is very regular and has been followed through six complete cycles. (c) shows the evolution of zonal kinetic energy (ZKE) and eddy kinetic energy (EKE) during one cycle. EKE is plotted against ZKE with time t as parameter; crosses are marked on the trajectory at 40 s intervals, and the arrows indicate the sense in which it is described. If the vacillation consisted solely in exchange of energy between EKE and ZKE while their total remained constant, the trajectory would simply be a straight line of slope −1. The departure from this form demonstrates the importance of baroclinic energy conversions, and other sources and sinks of total kinetic energy, during the vacillation cycle. EKE and ZKE are here defined as the true quantities divided by the total mass of the fluid.

the flow (see above). It is also a fundamental measure of the thermodynamic activity of the system (see Hide and Mason 1975). It is probably the most important single property of the system and of attempts to simulate it.

Fig. 2 summarizes a large number of laboratory and numerical results on the variation of heat flux with rotation rate in axisymmetric flow; the heat flux is scaled by a conductive value to give a 'Nusselt number'. Measured heat fluxes have been compared with modelled values for rotation rates (Ω) of zero, 0.1,...,0.5 rad s^{-1}. (Near $\Omega = 0.6$ rad s^{-1} the laboratory flow becomes non-axisymmetric.) The laboratory

Table I. *Some results from the steady wave comparison described by Hignett et al. (1985)*

	Laboratory measurement	Numerical simulation
Main-wave temperature amplitude (°C)*	0.27	0.28
Main-wave phase speed (rad s^{-1})†	7.8×10^{-3}	7.9×10^{-3}
Maximum zonal mean flow speed (cm s^{-1})	0.26	0.23

* Zonal wave 3 at mid-radius and mid-height

† Zonal wave 3; from velocity measurements

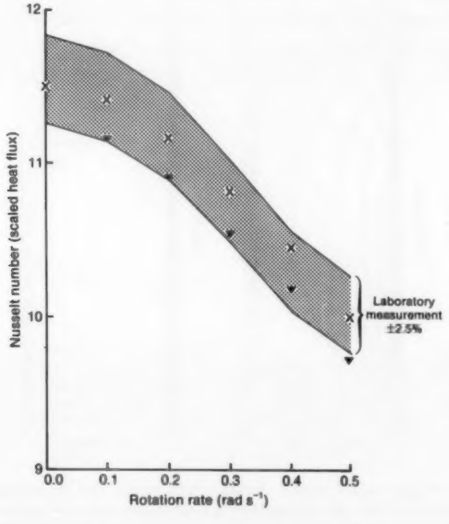

Figure 2. Measured and simulated total heat flux as a function of rotation rate in axisymmetric flow. The stippled area indicates the laboratory measurements ±2.5% error, low-resolution numerical model results are shown by crosses, and extrapolation to infinite resolution by solid triangles (see text for further details). The heat flux is plotted as a Nusselt number: the actual total heat flux divided by the flux which would be carried — under the same external conditions — by a solid having the same thermal properties as the working fluid. Apparatus geometry as in Hignett *et al.* (1985); imposed temperature difference 4 °C.

measurements are ascribed an accuracy (to random errors) of ±2.5%, and this range of values is indicated by the stippled area on Fig. 2. The general decrease of the heat flux with increasing rotation rate is well understood in terms of the concomitant decrease of the Ekman-layer thickness — see Hide and Mason (1975). The crosses on Fig. 2 show numerical model results obtained with 16 × 16 grid resolution. Integrations were also carried out with 24 × 24 and then 32 × 32 grid resolution at each rotation rate. Each set of three simulated heat fluxes (except the $\Omega = 0$ set — see below) was then extrapolated to notional infinite resolution using the power law method of de Vahl Davis (1983). The resulting values are shown as triangles on Fig. 2.

At rotation rates of 0.1 rad s^{-1} and above, both the low resolution and the extrapolated results lie within or extremely close to the range of the measured values (given the 2.5% error). It will be observed that the low-resolution results are generally closer to the actual measurements than are the extrapolated results. No unique explanation for this is offered. The numerical simulations may be converging to a value slightly different from the true one; or there may be some small systematic error in the laboratory measurements. Nevertheless, when all is said and done, the level of agreement shown in Fig. 2 is very good: the measured and simulated heat fluxes differ by less than 3%.

4.2 *Spurious eddy motion*

Fig. 2 shows no infinite resolution extrapolation at $\Omega = 0$. Considerable difficulties arose in this case. At 16 × 16 resolution good results were obtained (as Fig. 2 shows) but at 24 × 24 and 32 × 32 resolution time-dependent eddy motion was produced in and near the inner-cylinder's thermal boundary layer. An example of this behaviour is shown in Fig. 3(a), which is a snapshot of the streamfunction field; several

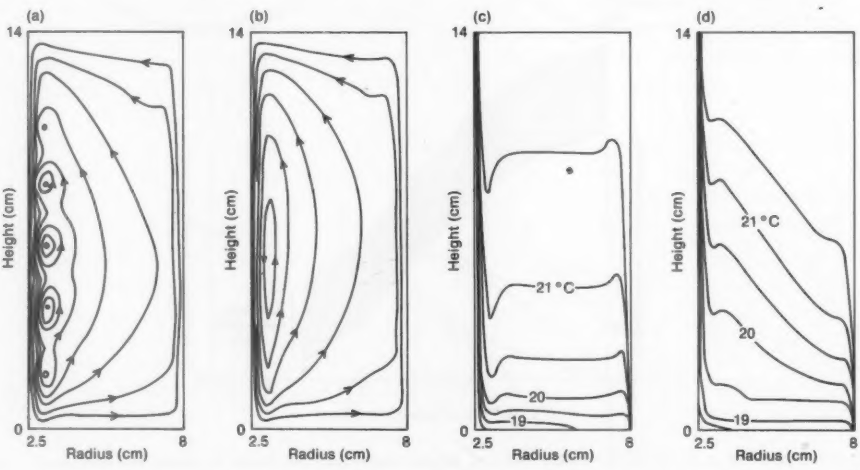

Figure 3. Cross-sections (in the height/radius plane) showing streamlines and temperature fields obtained in numerical simulations of axisymmetric annulus flow. (a) Instantaneous streamlines from an integration at zero rotation rate which developed spurious eddy motion in and near the inner cylinder; 32 × 32 stretched grid. (b) Steady-state streamlines from a well-behaved integration at zero rotation rate; 64 × 64 unstretched grid. (In both (a) and (b) streamfunction contours are plotted, and the arrows indicate the direction of flow; contour interval is the same in both cases.) (c) Steady-state temperature field corresponding to (b). (d) Steady-state temperature field obtained with a 32 × 32 stretched grid (as for (a)) but at a rotation rate of 0.5 rad s^{-1}. In all cases the inner and outer cylinder temperatures are 17 and 21°C, and the assumed geometry is as in Hignett *et al.* (1985).

closed circulations are seen. These eddies are spurious in that they do not appear in the real system under the conditions applied (as simple dye-injection experiments reveal). One of their effects is to increase the total heat flux by about 10% — well above the range of the measured values.

Figs 3(b) and 3(c) show the streamfunction and temperature fields in a well-behaved integration at zero rotation rate. It is clear, qualitatively, that the inner-cylinder thermal boundary-layer has the potential for instability, there being large horizontal temperature gradients as well as velocity gradients. Theoretical studies (Gill 1966) indeed show that instabilities should be expected under conditions more extreme than those applied here. What appears to be happening is that an increase in grid resolution is leading to a spurious lowering of the stability threshold. Numerous changes of finite difference scheme were tried but at the above resolutions all such variants of the model gave spurious eddies when the rotation rate was lower than 0.1 rad s^{-1} or so.

Can the effect be overcome by using still finer resolution? To investigate this, a series of integrations using unstretched $N \times N$ grids, with $N = 24, 32, 40, 48, 56$ and 64 has been carried out. Spurious eddy motion occurs at $N = 40$, but all the other cases give smooth fields. Fig. 4 shows the simulated heat fluxes plotted against the resolution N; the range of the laboratory measurements is also shown. Evidently the fluxes at high resolution are converging satisfactorily into the measurement 'corridor' (though integrations at even higher resolutions would be needed to confirm this). The occurrence of spurious eddies does indeed appear to be a pathological feature associated with intermediate resolution.

No detailed theoretical treatment of the phenomenon has yet been produced, but precedents are known. Bell and White (1987a) have established that short-wave baroclinic instabilities may be subject to spurious growth in quasi-geostrophic models having intermediate (~5 to 20 level) vertical resolution, while at lower resolutions they do not appear at all. Something similar concerning boundary-layer instabilities could well be happening at intermediate resolutions in the annulus model.

It might be asked why the problem of spurious eddy motion does not occur at higher rotation rates. The reason (qualitatively) is that rotation operates in several ways to stabilize the side boundary layers. It stabilizes directly through its 'stiffening' effect (the Proudman–Taylor effect — see Hide 1966, 1977

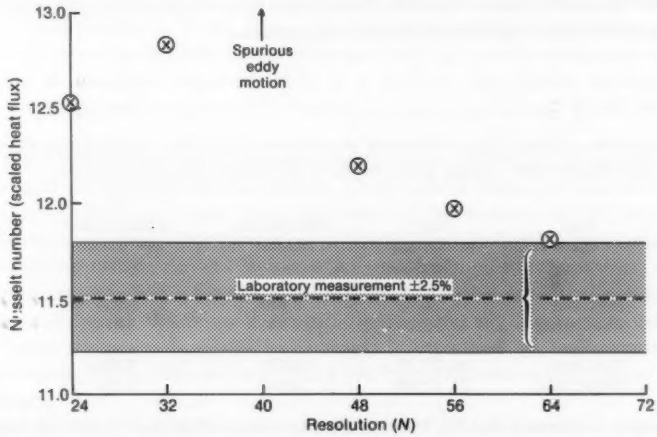

Figure 4. Variation of the steady-state heat flux with grid resolution N at zero rotation rate. Heat flux is expressed non-dimensionally as a Nusselt number — see caption to Fig. 2. Number of grid points is N (radial) $\times$ N (vertical), their spacing in each direction being even. The stippled area indicates the measured value ±2.5% error.

and Chandrasekhar 1961) — and indirectly (i) by weakening the meridional circulation and (ii) through the reduction of the temperature difference across the side boundary layers which occurs as thermal wind balance is attained in the interior. Fig. 3(d) shows the steady-state temperature field obtained when $\Omega = 0.5$ rad s^{-1}; the contrast with Fig. 3(c) is clear.

5. 'Bench-mark' comparisons

Axisymmetric flow in a non-rotating wall-heated annulus is similar in many respects to a classical problem of fluid dynamics — that of convection in a cavity of rectangular cross-section (and infinite length) whose vertical walls are maintained at different temperatures. This problem is important in many engineering applications, from double glazing to nuclear reactor design. It has been used by de Vahl Davis and Jones (1983) as a basis for comparison of many different numerical models with so-called 'bench-mark' values. These values were obtained by extrapolation to infinite resolution of results from a model which the authors considered to be well-behaved and reliable. A square cavity was assumed.

By suppressing the geometric curvature factors in the axisymmetric annulus model it is possible to apply it to the cavity problem and to find how it rates against the bench-mark values. Table II summarizes the heat fluxes found in two separate comparisons. That the results are so very close is not entirely surprising since the model used by de Vahl Davis and Jones was a grid-point formulation similar in many respects to the Met O 21 model. Nevertheless, the comparison affords a useful consistency check. More important, it suggests that verification of the Met O 21 model against laboratory heat flux measurements (as described in section 3) might be considered as a provisional verification of de Vahl Davis and Jones's model and of the several models which performed well in their bench-mark comparisons. There is the clear possibility (as yet unexploited) of using annulus measurements to verify a wide range of numerical flow models used in various scientific and engineering applications.

No spurious eddy motion occurred at any resolution in any of the integrations against the bench-mark. This probably reflects the fact that most of the flows studied in Met O 21's laboratories are subjected to stronger thermal forcing (as measured by an appropriate Rayleigh number) than the flows chosen by de Vahl Davis and Jones. Annulus flows might prove to be a very stringent test-bed for the models which figured in the original bench-mark comparison!

Table II. *Comparison of heat fluxes obtained in axisymmetric flow integrations using the Met O 21 model with the bench-mark values given by de Vahl Davis and Jones (1983)*

Rayleigh number*	Grid type	Grid resolutions used	Heat flux at highest resolution†	Extrapolation to infinite resolution	Benchmark value
10^6	Stretched	16, 24, 32	8.940	8.820	8.817
	Unstretched	48, 56, 64	9.084	8.814	
10^5	Stretched	16, 24, 32	4.574	4.514	4.509
	Unstretched	24, 32, 64	4.564	4.519	

* The Rayleigh number is defined as $\beta g \Delta T d^3/k\nu$, where β = expansion coefficient, ΔT = imposed temperature difference between the vertical walls of the cavity, d = side of (square) cavity, ν = kinematic viscosity, k = thermal diffusivity and g the acceleration due to gravity. In the bench-mark tests $\nu/k = 0.71$.

† The heat flux is expressed in non-dimensional terms as a Nusselt number — see caption to Fig. 2.

6. Theoretical models of annulus flows

Space does not permit more than a cursory survey of the 'scientific investigation' aspect of the numerical modelling work (see section 2). Model integrations have been used in numerous studies whose main objective has been to develop understanding of the rich variety of flows which occur in the rotating annulus; for examples from the recent literature see Bell and White (1987b), Read (1985, 1987) and White (1986), which are concerned with the construction and evaluation of various flow models based on quasi-geostrophic dynamics (see Gill 1982). Here some comments will be given on the extent to which the various non-axisymmetric flow phenomena are understood theoretically.

The transition from axisymmetric to non-axisymmetric flow is quite well explained by Eady's baroclinic instability analysis. Eady waves are unstable only for non-dimensional wave numbers (p) greater than 2.399; here $p = Nd\pi / \Omega\lambda$, where N is the buoyancy frequency, d is the depth of the fluid and λ is the dimensional horizontal wavelength of the wave. Thus, in any given annulus (and with specified values of N and Ω) the minimum possible value of p may be greater than 2.399 and so no instability occurs: the unstable waves may be too big to fit into the annulus. When the rotation rate Ω is increased, the value of p (see above definition) is reduced for a given dimensional wavelength, and instability eventually becomes possible. Calculated locations of the upper transition are in good order-of-magnitude agreement with the observed location. (A stronger statement is not supportable because the theoretical stability threshold is sensitive to the assumed zonal flow structure — see Bell and White 1987b.) To account for the lower transition it is necessary to invoke viscous and conductive effects; Hide and Mason (1975) review the relevant instability calculations.

Theoretical explanation of the regular regime is in poorer shape. Much work has been done by applied mathematicians on 'weakly non-linear theories'. These attempt to describe analytically the non-linear behaviour of weakly unstable waves: under various circumstances steady, vacillating or irregular wave solutions occur (see Hart 1979 for a review). Weakly non-linear treatments are monuments to the algebraic tenacity of their creators and they appear to account at least qualitatively for observed flows in the mechanically driven 2-layer system (see Read 1988), but their relevance to the thermal annulus is open to question. Fig. 5 shows the zonal mean flow in a typical simulated steady wave and in axisymmetric flow at the same rotation rate. It seems unlikely that any weak interaction theory can account for the radical difference in structure. Also, the theories predict amplitude vacillation at high rotation rates, whereas in the thermal annulus it occurs near the lowest rotation rate at which a given wave number can be sustained (Hignett 1985). It is possible that weakly non-linear theories are applicable, but that their usual manifestations assume an inappropriate marginally stable flow (White 1986).

Read (1985, 1987) and White (1986) have examined free-mode models for steady wave flow in the internally-heated and wall-heated annulus systems. These models have some success in accounting for the gross structure of the zonal mean flow and some of the features of the waves, but they are unsatisfactory in other respects. Representation of forcing and dissipative processes is desirable but mathematically difficult; Read *et al.* (1986) have developed a diagnostic framework.

The transition to irregular flow is broadly accounted for by the theory of Rossby wave instability (see Grotjahn 1984 for references). Behaviour in irregular flow is generally in accord with the predictions of geostrophic turbulence theory as developed by Charney (1971); indeed, the tendency of annulus flows to adopt internal jet form (White 1986, Bell and White 1987b) makes them a particularly suitable test-bed for Charney's theory.

A weak point in nearly all theoretical treatments of non-axisymmetric annulus flow is the representation of side boundary layers (see Hide 1968). The usual procedure is virtually to ignore them, but it may be that they play an important role in some time-dependent phenomena (such as amplitude vacillation).

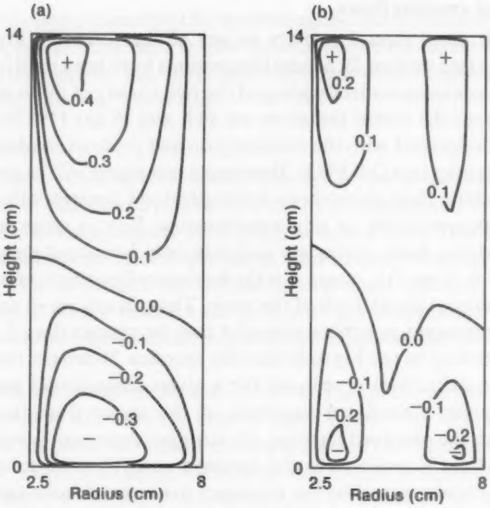

Figure 5. Height/radius cross-sections of the steady-state mean zonal flow in (a) axisymmetric and (b) non-axisymmetric numerical model integrations at the same rotation rate (1.0 rad s^{-1}). In this case the non-axisymmetric model produces a steady-wave flow, and the axisymmetric model (which, of course, does not allow non-axisymmetric motion) attains an entirely different steady state. The mean zonal flow structure seen in (b) is typical of steady-wave states. Units are cm s^{-1}. Assumed geometry as in Hignett *et al.* (1985).

7. Closing remarks

Laboratory systems are good testing grounds for numerical-flow models because the forcing processes which operate in them are physically simple and may be represented accurately using well-known formulae. In the atmosphere, on the other hand, the processes forcing the motion are complicated and difficult to represent reliably. Comparisons of real and simulated annulus flows have been described here; the results reveal generally good numerical-model performance, although some interesting spurious instabilities have come to light. Laboratory annulus measurements could be used to test and 'verify' numerical formulations in a wide range of other scientific and engineering applications.

Good numerical simulations provide a wealth of data which may be exploited in the development of theoretical and conceptual models of the flows. A brief assessment of the present state of understanding of non-axisymmetric annulus flows has also been offered in this paper.

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Books received

The listing of books under this heading does not preclude a review in the Meteorological Magazine at a later date.

The atmosphere — a wall chart, by K. Foley (Amsterdam, Mirage Publishing, 1987. Dfl.35.00, US $17.50) depicts the major phenomena responsible for meteorological events. The numerous sections are in colour with their associated explanatory notes.

Geophysical fluid dynamics, second edition, by Joseph Pedlosky (New York, Heidelberg, Berlin, London, Paris, Tokyo, Springer-Verlag, 1987. Price DM 160.00) is a high-level, unified treatment of the theory of the dynamics of large-scale motions of the oceans and atmosphere. Included are discussions on the fundamentals of geostrophic turbulence, thermocline theory and finite amplitude barocline instability.

General circulation of the ocean, edited by H.D.I. Abarbanel and W.R. Young (New York, Berlin, Heidelberg, London, Paris, Tokyo, Springer-Verlag, 1987. Price DM 160.00) reviews the new concepts and models put forward during the last 5 years, as well as the classical theories and observations. Avenues for future study are suggested, and it is aimed at advanced students and researchers in physical oceanography, meteorology and geophysical fluid dynamics.

Satellite photograph — 16 October 1987 at 0820 GMT

This NOAA-10 infra-red image shows cloud within the circulation of an intense depression centred near 55° N, 0° W. Cyclonically curved cloud bands surround the vortex. The variation in grey shades and the generally 'smooth' appearance of the cloud indicate that a series of cloud layers is present. However, at the vortex centre a small dark area is seen, which must be either cloud free or contain only low cloud. Comparison with visible imagery (not shown) indicates the latter to be true.

South-west of the British Isles, vigorous convection is occurring due to cold air within the circulation of the low having been swept southwards over the warm sea. The cloud band extending from Norway to Spain is associated with an eastward-moving cold front.

Photograph by courtesy of University of Dundee

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